The Pospieszalski Noise Model and the Imaginary Part of the Optimum Noise Source Impedance of Extrinsic or Packaged FET's

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***Abstract*— The imaginary part $X_{S_{\rm opt}}$ of the optimum noise impedance for extrinsic or packaged devices is investigated. The analysis modifies the well-known Pospieszalski noise model by applying a series feedback to the source port. A simple expression for $X_{S_{\rm opt}}$ is developed and is verified for extrinsic and packaged devices with a decreasing level of accuracy. The results give further insights into the way the parasitic inductors L_g and L_s affect the noise performance of the transistor and can help to design low-noise amplifier with simultaneous signal and noise power match at the input port.**

***Index Terms*—Amplifier noise, circuit modeling, feedback circuits, microwave FET amplifiers, semiconductor device noise.**

I. Introduction

In 1989, Pospieszalski [1] proposed a simple noise model for active devices such as MESFET's or HEMT's. Many researchers have granted experimental validity to this model throughout the years [2]–[4]. Hughes based on it an extensive investigation of the HEMT's noise behavior and proved that the noise figure can be predicted easily [5]. He also applied this model to a wide range of previously published devices [3] and showed that the noise equivalent temperature $T_{\rm ds}$ of the drain-source resistance ranges around 500 K for the extrinsic device while the intrinsic device can be modeled similarly with $T_{\rm ds}$ around 2000 K.

The Pospieszalski model has been applied to extrinsic devices because of its simplicity. However, this causes the model to fail to predict other noise parameters when considering extrinsic or packaged devices. This paper proposes a simple change in the Pospieszalski noise model as Hughes applied it in [3] in order to explain the behavior of $X_{S_{\rm opt}}$.

II. Analysis

The Pospieszalski noise model of the intrinsic device can easily be described with a $\mathbf{H}$ matrix because the noise sources $T_{\rm gs}$ and $T_{\rm ds}$ associated with $R_{\rm gs}$ and $R_{\rm ds}$, respectively, are uncorrelated [1], [6]. In order to improve the model when it is applied to either extrinsic [3] or packaged devices [7], a feedback element is added—the lossy source inductance L_s (Fig. 1). The circuit model is now a feedback network and can be easily analyzed as a particular case of [8]: the parallel feedback admittance is set to zero and the series feedback impedance is $Z_s = R_s + jX_s$ where $X_s = j\,2\pi f L_s$ and $R_s = \Re\mathrm{e}[Z_s]$ is source of thermal noise. After transforming the $\mathbf{H}$ representation into its $\mathbf{T}$ matrix representation and developing the noise parameters R_n, g_n, and ρ_n as functions of the model components, the optimum noise impedance $Z_{S_{\rm opt}}$ can be obtained. The final expression for $X_{S_{\rm opt}}$ is

$$\frac{X_{S_{\rm opt}}}{Z_o} = \frac{f_t}{f}\,\frac{x_{S_{\rm opt}}^{(e)} + \Delta x_n}{1 + \Delta x_d} \tag{1}$$

where

$$x_{S_{\rm opt}}^{(e)} = \frac{1}{g_m Z_o} - \frac{f}{f_t} x_s$$
$$\Delta x_n = \frac{1}{T_{{\rm ds}_o}}\left(1 + \frac{1}{g_m Z_o r_{\rm ds}}\,\frac{x_s}{Q_s}\right)$$
$$\Delta x_d = \frac{1}{r_{\rm ds} T_{{\rm ds}_o}}\,\frac{x_s}{Q_s}.$$

There, $Q_s = \Im\mathrm{m}[Z_s]/\Re\mathrm{e}[Z_s]$ is the Q of the inductor, $f_t = g_m/(2\pi C_{\rm gs})$ is the frequency where the short circuit gain is unity, $x_s = X_s/Z_o$ is the reactive series feedback value normalized to the characteristic impedance Z_o, and $r_{\rm ds} = R_{\rm ds}/Z_o$, $T_{{\rm ds}_o} = T_{\rm ds}/T_o$, $T_o = 290$ K are normalized values of elements of the model in Fig. 1. Notice that $R_{\rm gs}$ does not appear in (1). The remaining noise parameters can be worked out similarly but their expansions give rise to much more involved expressions [1].

At the frequency $f = \omega/(2\pi)$, (1) can be simplified to

$$X_{S_{\rm opt}} \simeq \frac{1}{\omega C_{\rm gs}} - \omega L_s \tag{2}$$

if

$$\begin{cases} \Delta x_n & < x_{S_{\rm opt}}^{(e)} \\ \Delta x_d & < 1 \end{cases}$$

is verified. This approximation is a very simple expression whose implications are now developed.

III. Validation

Expression (2) is dependent only on $C_{\rm gs}$ and L_s at the frequency $\omega/(2\pi)$ for the case of either a lossy or a lossless inductor L_s. The result is valid for this model as long as $T_{\rm ds}$ and $R_{\rm ds}$ are large; the exact value of $T_{\rm ds}$ is not really important for the determination of $X_{S_{\rm opt}}$. Hughes has proved that this

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TABLE I
COMPARISON BETWEEN MODELS CITED IN [3] WITH $T_{\rm ds} = 2000$ K. EVERY MODEL HAS BEEN EVALUATED AT THE TOP END OF ITS FREQUENCY RANGE. $C_{\rm gs}, L_s, L_g$, and R_s ARE THE VALUES AS GIVEN IN EACH REFERENCE

Reference	Range GHz	R_s Ω	C_{gs} fF	L_s pH	L_g pH	f_{max}/f_t	$\Delta x_n/x^{(e)}_{S_{opt}}$	Δx_d $\times 10^{-5}$	C_{opt} fF	L_{opt} pH
[2]	4–18	0.70	224	6.59	42.4	0.4160	0.4631	1.348	259.90	46.78
[9] (FET)	11–13	1.67	300	0	0	0.8168	0.2242	1.422	314.20	0.37
[9] (HEMT)	11–13	3.37	250	0	0	0.3713	0.4118	4.017	259.90	0.64
[16]	40–60	2.20	76	25	10	0.9550	0.3071	1.380	93.85	31.28
[17]	2–18	3.50	270	23	340	0.3563	0.6963	5.006	248.90	362.00
[18]	2–18	3.20	240	30	50	0.6169	0.3626	2.861	239.40	84.59
[19]	12–25	2.98	127	38.6	147	0.9684	0.1751	0.955	129.30	183.80
[20]	1–25.5	2.72	96.4	5	51.6	0.1236	0.9478	9.646	79.51	53.50

is true for $T_{\rm ds}$ [3]. $R_{\rm ds}$ is usually in the range of hundreds of ohms.

In order to validate (2), some references used in [3] have been analyzed (Table I). The references provide a complete list of the values of the components. It is worth pointing out that these models as presented in those papers have been optimized for matching the measured S parameters in a given frequency range. As Hughes highlighted, noise figure and associated gain are often the only published quantities available for characterizing the noise performance. Table I has been developed according to this procedure. The room temperature has been assumed to be $T_{\rm room} = 298$ K; the input resistance $R_{\rm gs}$ has an equivalent noise temperature $T_{\rm gs} = T_{\rm room}$ for quite a large spread of the drain current $I_{\rm ds}$ [2]; the equivalent noise temperature $T_{\rm ds}$ of the output resistance $R_{\rm ds}$ has been set to $T_{\rm ds} = 2000$ K, as [3] suggests. In [2] (not cited in [3]), the simulation has been carried out with $T_{\rm ds} = 2550$ K but the results in Table I for $x^{(e)}_{S_{\rm opt}}/\Delta x_n$ and Δx_d refer to $T_{\rm ds} = 2000$ K. The value $T_{\rm ds} = 2000$ K has been chosen for the analysis because the topology of the device models is available and $R_{\rm ds}$ is therefore part of the intrinsic device embedded within the external components. Reference [9] outlines one MESFET model and one HEMT model; they consist of resistive and capacitive elements only.

The frequency dependence of the optimum noise reactance has been approximated with a least squares fit for each reference of Table I with an expression similar to (2)

$$X^{(i)}_{S_{\rm opt}} = \frac{1}{\omega_i}\frac{1}{C_{\rm opt}} - \omega_i L_{\rm opt}. \tag{3}$$

The models provided by the references of Table I have been used to determine $X^{(i)}_{S_{\rm opt}}$ for each angular frequency ω_i with a circuit simulator; the frequency range $(\omega_i, i = 1\ldots N)$ varies according to the published reference (Table I). The least squares fit (3) has been applied to packaged devices [10] with some considerable degree of agreement (Fig. 2).

Equation (3) proves that

1) the expression (2) fits the data of the device circuit model;
2) a simple Pospieszalski noise model with feedback can successfully be applied to simulate $X_{S_{\rm opt}}$ of extrinsic or packaged devices.

By comparing the values for $C_{\rm opt}$ and $L_{\rm opt}$ to $C_{\rm gs}$ and L_s respectively (Table I), it is clear that $C_{\rm opt} \simeq C_{\rm gs}$ while the inductance $L_{\rm opt} \neq L_s$. Others [11] have confirmed that the Pospieszalki noise model for the intrinsic device provides $X_{S_{\rm opt}} = 1/(\omega C_{\rm gs})$. In fact, for the Pospieszalki noise model (intrinsic), $X_{S_{\rm opt}} = -\Im{\rm m}[Z_{\rm in}]$ where $Z_{\rm in} = R_{\rm gs} + 1/(j\omega C_{\rm gs})$ is the input impedance. This observation suggests that if a series inductor L_g is connected between the source and the gate input, then $-X_{S_{\rm opt}} = \Im{\rm m}[Z_{\rm in}] = \omega L_g - 1/(\omega C_{\rm gs})$ or more generally as a first approximation, that $-X_{S_{\rm opt}}$ is the sum of the reactive components through which the current from the input port flows (Fig. 3). Therefore, (2) is modified

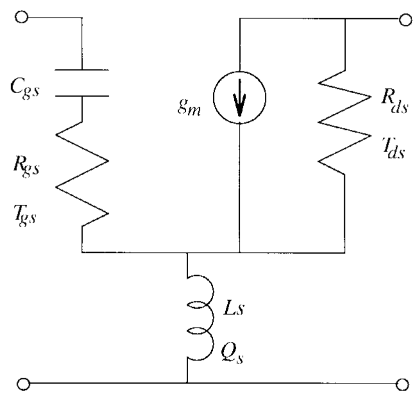

Fig. 1. Noise model for extrinsic or packaged devices. The feedback is a source of thermal noise if Q_s is specified.

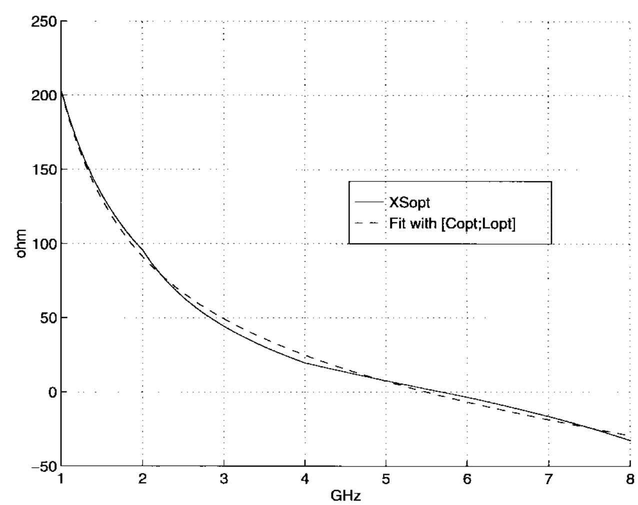

Fig. 2. Comparison between (3) and the simulated $X_{S_{\rm opt}}$ for Hewlett-Packard ATF10136.

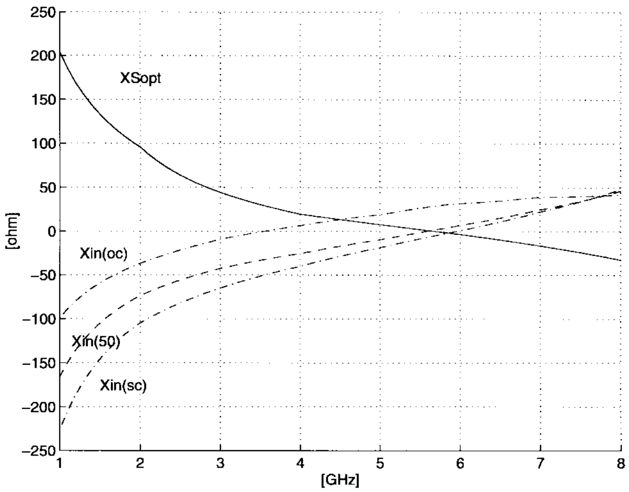

Fig. 3. Simulated imaginary part of $Z_{S_{\rm opt}}$ and $Z_{\rm in}$ for Hewlett-Packard ATF10136 when the output port is terminated by an open circuit (oc). $Z_0 = 50\,\Omega$ (50) or a short circuit (sc).

accordingly:

$$X_{S_{\rm opt}} \simeq \frac{1}{\omega C_{\rm gs}} - \omega(L_s + L_g) \tag{4}$$

Table I shows that excellent agreement is achieved.

Equation (4) improves the understanding on how parasitic inductances affect the noise performance of the device [12], [13]. L_g is not in the feedback branch and will not have the same effects on device performance as L_s [14], [15]. The two inductors L_g and L_s give the designer the freedom to set $X_{S_{\rm opt}} = 0$ at the frequency where $R_{S_{\rm opt}} = 50\ \Omega$. Explicit equations for $R_{S_{\rm opt}}$ and $X_{S_{\rm opt}}$ allow the designer of either monolithic microwave integrated circuit (MMIC) or surface mounted low-noise amplifiers (LNA's) to determine, at a given frequency, the values of external inductors that provide $R_{S_{\rm opt}} = 50\ \Omega$ and $X_{S_{\rm opt}} = 0$. The MMIC designer has one more degree of freedom because of the ability to control $C_{\rm gs}$.

IV. Conclusion

A simple equation explains the behavior of the imaginary part of the optimum source impedance for minimum noise figure $Z_{S_{\rm opt}}$ of extrinsic or packaged transistors. The result is shown to be consistent with different circuit models previously published and it is based on the widely accepted Pospieszalski noise model. The expression confirms that the Pospieszalski noise model developed for intrinsic devices is well suited for extrinsic as well as packaged transistor for a quick investigation of their noise performance and for design purposes.

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